Bass Reflex Performance Envelope

This White Paper takes a novel approach to estimating the performance envelope available from ported loudspeakers. The fourth order Butterworth high pass response is assumed. Relationships are established between port and driver sizes, system f3, maximum system SPL, box size, and the required input power. It is shown that for some desirable solutions a port can limit the performance, and a passive radiator is suggested instead. A series of Excel spreadsheets accompany the paper to allow the reader to experiment with different design options.

Gary Gesellchen
Vanatoo LLC
Seattle, WA
February 2020

Vanatoo
Technology Serving Art

Preface

This is a White Paper, not a midterm exam. While attempting to be factually correct, it is intentionally written in a conversational style. You probably need to be an engineer to dig into the underlying math and challenge the assumptions, but the intent is that you do NOT need to be an engineer to understand the conclusions and the point of it all. If you know what "frequency response" means, and you know that all devices have physical limits, you should be able to follow the argument and understand the conclusions.

Abstract

When confronted with a clean sheet, the loudspeaker designer must ask themselves "what's possible?" While overall sound quality is what defines a system, the bass response is typically what dictates the size, power requirements, and a good deal of the cost of a loudspeaker system. This paper will look at a subset of designs typically possible from a "bass reflex" loudspeaker. The results can be used by the designer to quickly home in on what's possible, and to define the requirements for a proposed system. The results can also be used to "reverse engineer" an existing design so as to know what to expect. This paper also shows which bass reflex designs can be realized with a ported cabinet versus the family of very attractive design options that are only realizable with a passive radiator design.

Introduction

"Bass Reflex" is sort of an old fashioned term that was applied to what today we more typically call ported or vented speakers. I've chosen to use the term here since I think it evokes the sort of working action shared by both ported and passive radiator speaker systems, and can be used as a single term to cover both. A bass reflex system is one in which another moving element is added to the speaker enclosure in addition to the driver. This element, be it the mass of air in a port or the diaphragm of a passive radiator, is driven by the air pressure in the box. When properly constructed the additional moving element will resonate at a lower frequency than the driver and effectively help extend the system's bass response. While "resonance" is typically considered a dirty word in audio, a well-designed bass reflex system can have tight, articulate bass that is very satisfying to even the most critical listener.

There seems to be a lot of misinformation on how a bass reflex system works. I often read about how it utilizes the sound coming off the backside of the woofer cone to increase the bass output. While this is sort of true, it really blurs the physics of what is happening. A brief description that technically holds water seems in order.

A ported system is one that typically has a tube inserted in the cabinet such that air can flow in and out. Sometimes a slot is used instead of a tube, and for the most part the exact shape of the opening is of secondary importance. The important thing is that there is a slug of air that is defined by the dimensions of the port, and this slug of air has a known mass. When the air in the port moves into the box it compresses the air that is already inside, increasing the pressure, which wants to force the slug of air back out of the box. The compressed air in the box therefore represents a "stiffness", or spring. In classic engineering mechanics the two requirements for a resonance are a mass and a stiffness. Our system has both, and therefore the mass of air in the port will resonate against the stiffness of the air in the box, resulting in what is known as a "Helmholtz resonance". This resonance is something you probably experienced at a young age when you blew across the top of a bottle and were able to generate a steady tone.

The Helmholtz resonance of the port and box is what a bass reflex speaker uses to extend the bass a speaker might otherwise possess. The motion of the driver in-and-out produces pressure in the box, which in turn drives air in-and-out of the box through the port. The in-and-out motion of the driver produces sound, and the in-and-out motion of the air in the port does too. The visualization of how these add together in our system is shown later in the paper, but the design trick is to get these two sound producing mechanisms to blend together seamlessly.

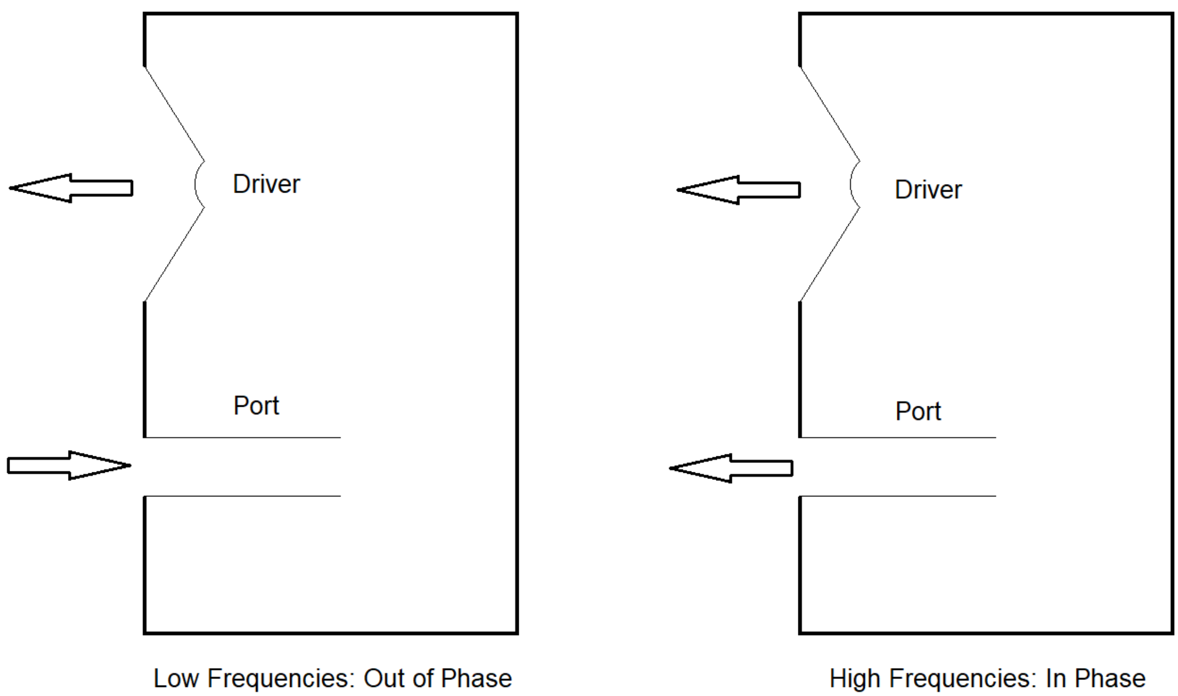

Figure 1: Ported bass reflex speaker, motion of driver and air in port air at low and high frequencies

Figure 1 shows the cross-section of a ported bass reflex. At low frequencies the outward motion of the driver creates a vacuum in the box, drawing air in through the port. As the frequency increases to the Helmholtz resonance and above, the moving air in the port can no longer keep up and starts to lag the cone motion. At higher frequencies the air in port has enough inertia that it moves very little, and the lag is great enough that it is effectively in phase with the motion of the driver.

Thuras was issued the original patent for this type of system in 1932. It wasn't until the 1960's (Thiele) and 1970's (Small) until the behavior of the system was thoroughly defined mathematically. The Thiele/Small model of bass reflex loudspeakers shows how to properly configure the driver, box, and port such that the overall sound generated by the port and the driver blends together in a predictable and desirable manner. The widespread adoption of the T/S model and the success of systems designed using it is responsible for the explosion of high quality (if often somewhat similar) ported systems on the market today. At this point it is worth noting that a passive radiator, which often just looks like a driver with no magnet or motor, works almost identically the same as a port. The above description describes its behavior too, and both can be considered "bass reflex".

Response Types

Thiele, Small, and most of the pioneers of loudspeaker design were electrical engineers. Different ways of filtering electrical signals was a hot topic in electrical engineering in the early 1900's, and a lot of different filters with different response types were developed and documented. Thiele recognized that a bass reflex speaker is analogous to an electrical high pass filter, and that if he looked at its behavior from the right mathematical perspective, the already well developed theory of electrical filters could be used to describe a speaker's behavior. Thiele defined different ported speaker "alignments" that would cause a speaker's response to match one of the already well known electrical filters, and generated a set of design tables that showed how to build such systems. A decade later Small extended Thiele's work to include boxes with losses, more alignments, and other really useful insights. While a speaker is not required to match any known filter, doing so meant you understood its behavior at a time when no engineer had a computer on their desk!

This paper will concentrate on just one of the infinite possible responses: The 4th Order Butterworth high-pass response (or "alignment"). The Butterworth response is chosen for several really good reasons. It is arguably "the best" of all possible responses because it typically has the lowest -3dB frequency of any system that has a "flat" frequency response without humps or ripples. The Butterworth design also requires that multiple design parameters be precisely related mathematically, which allows the analysis to be reduced to some really simple yet powerful relationships. While it might be possible to have higher output or lower f3 than this model predicts with a different "alignment", the results of the Butterworth analysis should be pretty indicative of bass reflex in general.

The Analysis Approach

The approach taken in this analysis pulls together a lot of different pieces from loudspeaker design and acoustics to tell its story. It starts with limiting the scope to systems with a Butterworth response. It shows that if you have a ported system with a Butterworth response, you don't need a lot of other constraints to really understand the system. If you know the system -3dB frequency (f3), you already know the frequencies that limit the maximum SPL the system can produce. If you know the size of the driver based on the currently popular sizes on the market ("4 inch", "8 inch", etc.), you can make a reasonable estimate of the system's maximum SPL. The port is defined as being big enough to keep up with what the driver can deliver in terms of SPL. The relationship between box size and power is developed, where it is shown that the designer is theoretically free to choose any box size. The box size and port that will work best for the chosen design are defined. Lastly the requirements that the design places on the driver are defined.

Assumptions

This analysis is going to apply the T/S model to a 4th Order Butterworth system (B4 from now on) and assume that the model can predict when the system reaches its physical limits. Experienced speaker designers will know that T/S is a "small signal" model, and is not generally considered valid at large amplitudes. There are newer models than T/S that include the effects of the speaker parameters varying with displacement and current in a nonlinear manner, but they are messy and complicated. We know that the linear T/S model isn't really valid at the larger amplitudes that we are going to encounter in this analysis, but it is better than throwing our hands up and saying we don't know anything. In the author's own experience the driver stiffness (and PR stiffness if one is employed) is the parameter that changes most with large excursions. Drivers tend to look about 50% as stiff when driven to their Xmax than they do when measured in the small-signal domain. Reducing the driver stiffness by 50% in the model shows that in most cases the overall results aren't changed dramatically. The maximum SPL is typically reduced by about 3dB when the stiffness drops by 50%, with all of the trends remaining the same. The sum of the nonlinear effects means the system won't really behave like our model at its limits, but the overall behavior will be close enough to make the analysis worthwhile.

An assumption of the T/S model is that the driver is mounted on an infinite wall, radiating into half-space. This becomes important when we get to the part of the analysis where we consider how much power is needed to drive the system.

We are going to assume specific relationships between a driver's advertised size and its cone area (Sd) and maximum usable displacement (Xmax). Neither one of these relationships will match many real world drivers exactly, but in the author's experience they represent good averages of what is currently available in high quality hi-fi drivers. We will also assume a few other details about our driver and port.

The Analysis

4th Order Butterworth High-Pass

The frequency response of a bass reflex speaker with a 4th order high pass Butterworth response and a f3 (-3dB frequency) of 100Hz is shown as Figure 2. In this example the system is putting out 83dB, and you can see it drops to 80dB, or -3dB, at 100Hz.

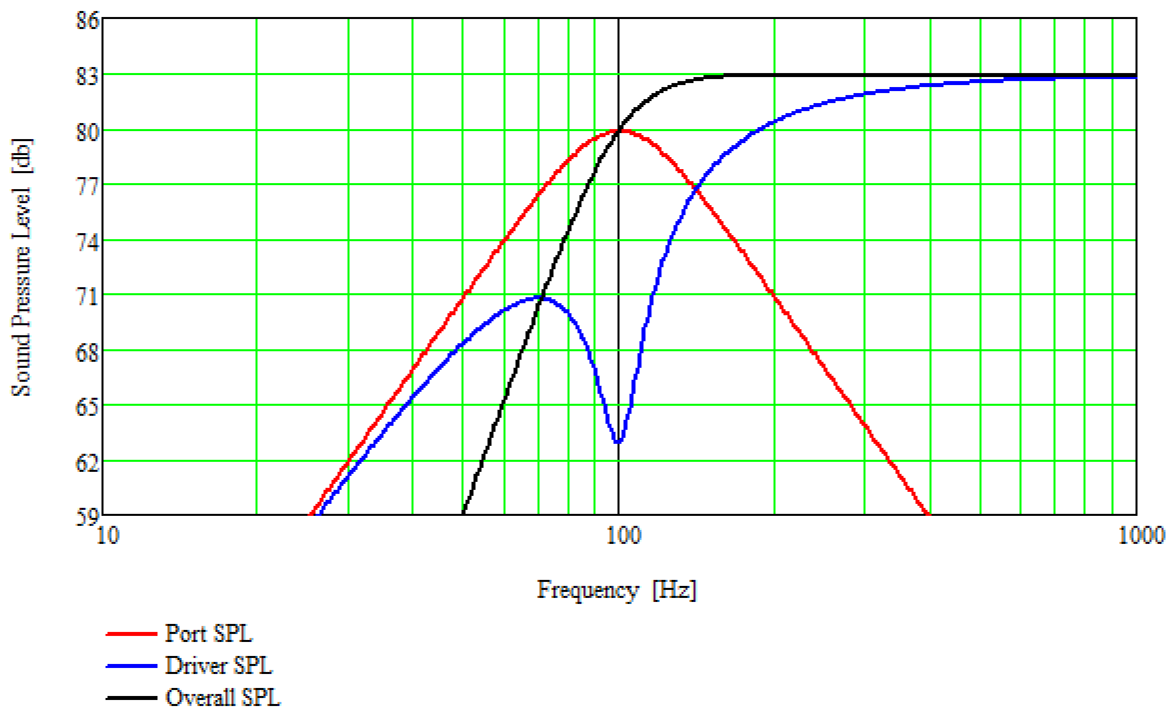

Figure 2: 4th Oder Butterworth Speaker Response

Figure 2 is a Butterworth frequency response, but all bass reflex alignments responses are similar in nature. Figure 1 showed us the basic behavior at very low and very high frequencies. Figure 2 shows us how those behaviors add together in the frequency domain.

- Below box resonance the port and driver are moving out of phase (see Figure 1), which makes their two outputs partially cancel one another. Therefore their sum, which is the system output, is less than either one individually. Individually each drop at a rate of 12 dB/Oct below the f3 frequency. But since they are out of phase, they sum to a response that drops at 24 dB/Octave, which is characteristic of "4th Order" (each "Order" is 6 dB/Oct).
- At box resonance we can see there is a sharp dip in the driver output, with nearly all of the system output coming from the port. The port and driver migrate from being mostly out of phase an octave below the box resonance to mostly in phase an octave above the box resonance.

- Above the box resonance the port and the driver are moving in phase (see Figure 1), but the port output decreases rapidly with increasing frequency (at 12dB/Oct), such that by the time you get to 2*f3 (200 Hz) the driver is putting out most of the sound.

Figure 3 shows the displacement of the driver and the air velocity in the port for the system with the response shown in Figure 2. These two parameters were chosen because they are what limit the physical output of the system. All real-world drivers have a physical limit of how far the cone can move in and out without damage. The distance they can travel while still being under good control of both the magnetic motor and the cone's suspension system will always be somewhat less, and is designated by the manufacturer as Xmax. Xmax is the farthest you can practically drive the cone before you run into excess distortion. Similarly, only so much air can flow through the port before it starts to resist the air flow and stops working properly. The variable that determines if the port will flow well is the velocity of the air in the port, which is why it is plotted in Figure 3.

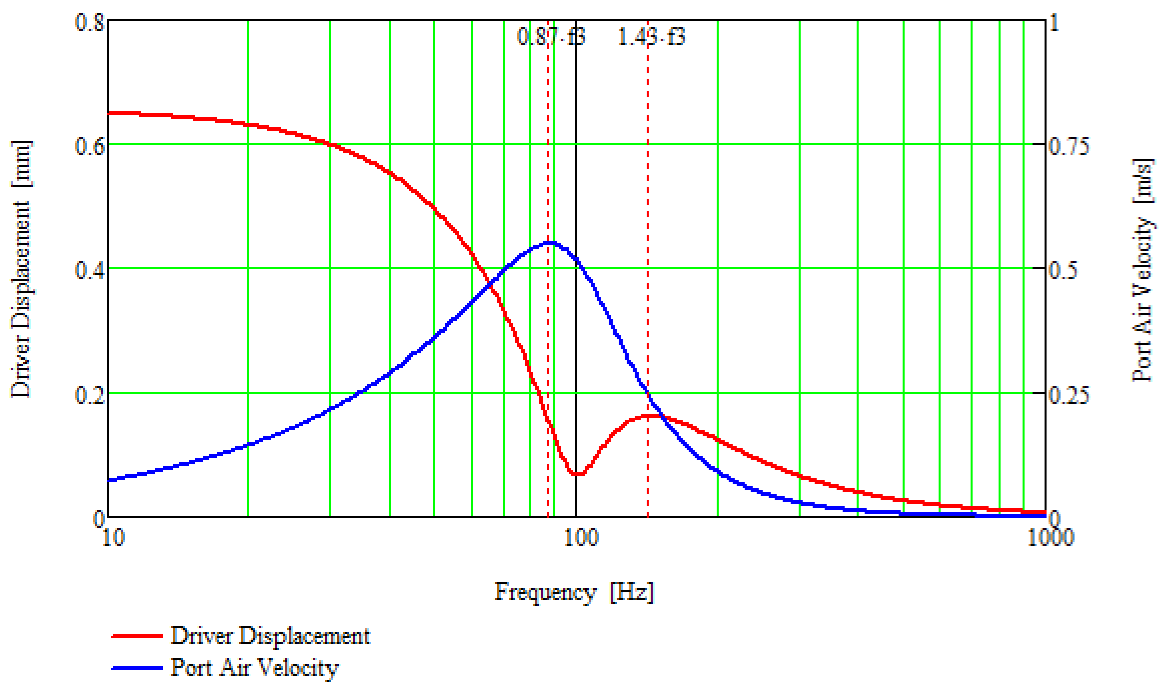

Figure 3: 4th Oder Butterworth Driver Displacement and Port Velocity

Looking first at the port velocity in Figure 3, we see that it peaks at 87Hz. Looking next at the driver displacement (above the 100Hz f3), we see that it peaks at 143Hz. The above trend will hold for all Butterworth ported designs: The port will max-out at 0.872*f3, and the driver will max-out at 1.432*f3. The curves will always look similar to Figure 3, with the frequency axis scaled by the actual f3 frequency. This might not seem like much, but its hugely important. It says that if we have a Butterworth response and we know the f3, we also know the 2 frequencies where the physical limits, driver displacement and port velocity, happen.

Note that the driver displacement increases radically below the f3 frequency and is limited at DC only by the driver stiffness. This is why ported speakers can experience cone flapping displacements when fed signals below their passband. We're going to assume that the displacement below the system f3 will be taken care of by some other means and that we only need to worry about the maxima above f3. Note also that the large decrease in driver displacement around the f3 is one of the big advantages of the ported vs a sealed design. Most of the distortion producing mechanisms in a driver are related to displacement, and producing more SPL for a given displacement generally means less distortion.

If we study Figure 2 at the above maxima frequencies we will see that at 0.87*f3 the port is down by 3.63dB compared to the system's mid-band. Similarly we see that the driver is down by 5.86dB at 1.43*f3. We now know the frequencies and something about the output levels that are going to represent the system limits.

$$fpmax(f3) = 0.872 * f3 \qquad \text{EQ(1)}$$

$$fdmax(f3) = 1.432 * f3 \qquad \text{EQ(2)}$$

$$SPLpm = -3.63 \qquad \text{(EQ3)}$$

$$SPLdm = -5.86 \qquad \text{(EQ(4)}$$

Where: fpmax is the frequency of max port air velocity
fdmax is the frequency of max driver displacement
SPLpm is dB the port is down compared to mid-band
SPLdm is dB the driver is down compared to mid-band

If we define the maximum system mid-band SPL as SPLM, we know that at the system limits:

$$SPLp(fpmax(f3), SPLM) = SPLp(0.872 * f3, SPLM) = SPLM + SPLpm = SPLM - 3.63 \qquad \text{EQ(5)}$$

$$SPLd(fdmax(f3), SPLM) = SPLd(1.432 * f3, SPLM) = SPLM + SPLdm = SPLM - 5.86 \qquad \text{EQ(6)}$$

Where: SPLp is the SPL produced by the port at one meter
SPLd is the SPL produced by the driver at one meter

Equations 5 and 6 are enough to fully constrain the system response based on the physical limits of the port and driver. They say what the port and driver must each be capable of doing to support a system with a given -3dB frequency (f3) and given maximum SPL (SPLM). At a frequency of 0.873*f3 the port must be able to output an SPL of SPLM-3.63dB, and at frequency 1.432*f3 the driver must be able to output an SPL of SPLM-5.86dB. We have constrained what the port and driver must be able to do to meet a given B4 response at a given maximum SPL, and it resulted in two pretty simple equations!

Driver and Port Output

In order to make use of Equations 5 and 6 we need to know how the physical limits of the port and driver affect the SPL they can produce. The T/S model assumes that the driver is mounted on an infinite plane. The equations for the sound radiated by a circular piston on an infinite plane are well established. The simplified version of the SPL produced at one meter on-axis from a circular piston on a plane is:

$$SPL(f,S,V) = 20 * log\left(\frac{\rho * f * S * V}{2 * 10^{-5}}\right) \qquad \text{EQ(7)}$$

Where: f is the frequency [Hz]
ρ is the air density of 1.204 [kg/m^3]
S is the piston area [m^2]
V is the piston velocity [m/s]

If we assume that the driver and port can be considered as flat circular pistons on an infinite plane, we can derive the SPL they generate at one meter on axis as shown.

$$SPLp(f,Sp,Vp) = 95.59 + 20 * \log(Sp * Vp * f) \qquad \text{(EQ(8)}$$

$$SPLd(f,Sd,Xd) = 111.56 + 20 * \log(Sd * Xd * f^2) \qquad \text{(EQ(9)}$$

Where: f is the frequency
SPLp is the SPL at one meter from the port [dB]
Sp is the port area [m^2]
Vp is the port air velocity [m/s]
SPLd is the SPL at one meter from the driver [dB]
Sd is the driver area [m^2]
Xd is the driver displacement [m]

If we set Equation 8 equal to Equation 5 we have the port constraint. Setting Equation 9 equal to Equation 6 constrains the driver.

$$SPLM - 3.63 = 95.59 + 20 * \log(Sp * Vpmax * f) \qquad \text{(EQ(10)}$$

$$SPLM - 5.86 = 111.56 + 20 * \log(Sd * Xdmax * f^2) \qquad \text{(EQ(11)}$$

Where: Vpmax is the maximum allowable port velocity
Xdmax is the driver's published Xmax

According to Equation 10 the port is limited by its area, the frequency, and the maximum velocity. We're going to assume without proof that a reasonable maximum velocity limit is 20 m/sec. This is a number that floats around the Audio DIY forums and seems to correlate well with the author's own experience.

If we assume Vpmax is 20, we can rearrange Equation 10 to solve for the port area (Sp), substitute in Equation 1 for the frequency, and simplify as:

$$Sp(f3, SPLM) = \frac{10^{((SPLM-124.05)/20)}}{f3} \qquad \text{EQ(12)}$$

Equation 12 is the area of the port [m^2] required to produce the SPL necessary for a system with a -3dB frequency of f3 to produce a maximum SPL of SPLM. The port is now adequately defined. Equation 12 will prove useful as shown, but can also be rearranged to solve for SPLM as a function of the port diameter (Dp) as shown in Equation 14.

$$Dp(Sp) = 44.424 * \sqrt{Sp} \qquad \text{EQ(13)}$$

Where: Sp is port area in m^2
Dp is port diameter in inches

$$SPLM(f3, Dp) = 58.14 + 20 * \log(f3 * Dp^2) \qquad \text{EQ(14)}$$

Where: SPLM is the system mid-band Max SPL in dB
F3 is the system -3dB frequency in Hz
Dp is port diameter in inches

(Note the mis-mash of units in Equation 13. This paper normally uses SI units, but port and driver sizes are typically specified in inches. We need Sp and Sd in meters squared for the other calculations.)

Equations 12 and 14 define the port requirement. Equation 11 will similarly be used to define the driver requirement to support f3 and SPLM. Looking at Equation 11 we see the sum of Sd*Xmax, which is defined as the driver volume displacement. We could leave Sd and Xmax in the equations as they are both readily available for high quality drivers. But we are interested in making sweeping system-level statements. It would be more useful if we could make a reasonable approximation of the quantity Sd*Xmax as a function of the driver size. This would tell us how big of driver is required for any combination of f3 and SPLM.

Commercial Drivers

Loudspeakers drivers are marketed in the US by their nominal size, specified as the driver frame outside diameter in inches. The diameter of the cone is always smaller than the advertised diameter, d. I took a look at the published cone area (Sd) of a lot of different drivers (ie: woofers) with different advertised sizes (d). While there is obviously some variation, the following relationship was found to closely match the parameters of commercially available drivers.

$$Sd(d) = 0.00035 * d^2 \qquad \text{EQ(15)}$$

Where: d is published diameter in inches
Sd is cone area in m^2

We can also make some generalizations about the driver Xmax as a function of the advertised size. The drivers we're concerned with are relatively long travel woofers. A reasonable approximation for the Xmax you can expect from a high quality woofer as a function of its advertised size is:

$$Xmax(d) = 2 + d/2 \qquad \text{EQ(16)}$$

Where: d is published diameter in inches
Xmax is the displacement in mm

Combining Equations 15 and 16 gives us the volume displacement (Sd*Xmax) as a function of the driver size:

$$VD(d) = 3.5 * 10^{-7} * (\frac{d^3}{2} + 2 * d^2) \qquad \text{EQ(17)}$$

Where: d is published diameter in inches
VD is the volume displacement in m^3

Figure 4 shows Equation 17 plotted as Hi-Fi Drivers. Subwoofer Drivers, with their emphasis on low frequency output, can typically have as much as twice the Xmax we assumed in Equation 16. Pro Audio Drivers, with their emphasis on efficiency, will typically have about half the Xmax we assumed in Equation 16. Both are plotted in Figure 4. They represent 6 dB more/less maximum output than our target Hi-Fi solution.

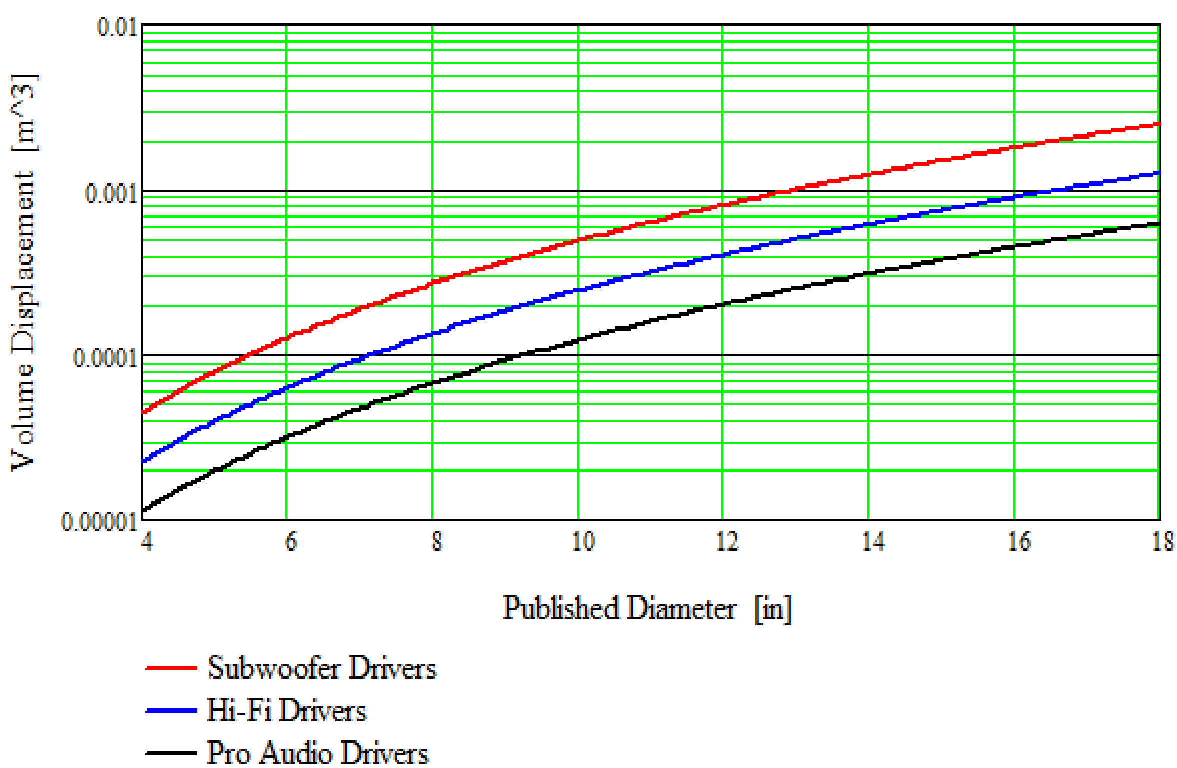

Figure 4: Driver Volume Displacement vs Size. Equation 17 is plotted as Hi-Fi Drivers.

Driver and Port Output (Cont'd)

We are now in a position to substitute Equation 17 for the volume displacement as a function of the driver size into Equation 11 for the driver requirement. Doing so gives the maximum system SPL driver requirement as:

$$SPLM(f3, d) = -5.46 + 20 * \log\left(f3^2 * \left(\frac{d^3}{2} + 2 * d^2\right)\right) \qquad \text{EQ(18)}$$

Equation 18 gives us our estimate of the maximum mid-band system SPL (SPLM) as a function of two really basic things: 1) The system -3db point (f3) 2) The advertised size of the driver. It makes perfect sense, yet it's still a great finding.

We now have 2 pretty simple equations for the maximum system mid-band SPL based on the system f3, the port diameter (Equation 14), and the driver size (Equation 18). Plotting Equations 14 and 18 against the system -3dB frequency (f3) will show us how the maximum mid-band SPL (SPLM) varies with the port and driver size.

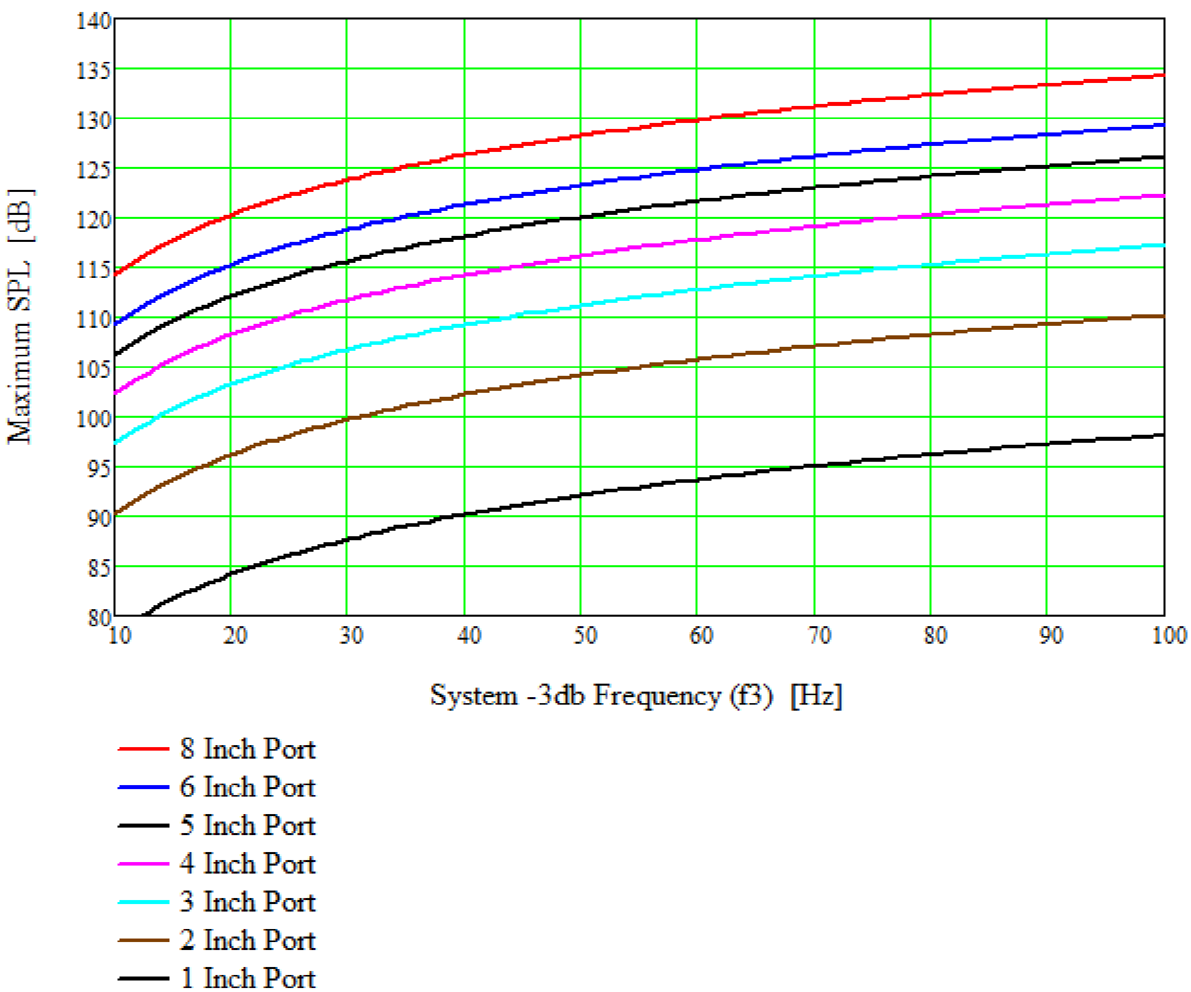

Figure 5: Maximum System SPL vs System f3 for various Port Diameters (from Equation 14)

Figure 5 is a plot of Equation 14 with the port based maximum SPL plotted against the system f3. This is all of a sudden a lot of information. Speaker system f3 frequencies are widely, if inaccurately, reported. Figure 5 says that if you know the system f3 and the port diameter, you know the maximum SPL it is likely to play cleanly. There's no guarantee that any commercially available system is designed according to our assumptions. But our assumptions have all tended toward best case, and since Figure 5 represents the maximum system SPL for our hypothetically prefect system, it likely represents a reasonable upper limit of expectations from a commercially available system.

Figure 5 says that if you want to play low and loud, you need a really big port. If the system f3 is reliably known, Figure 5 will put a reasonable upper limit on the maximum SPL one should expect based on the port size.

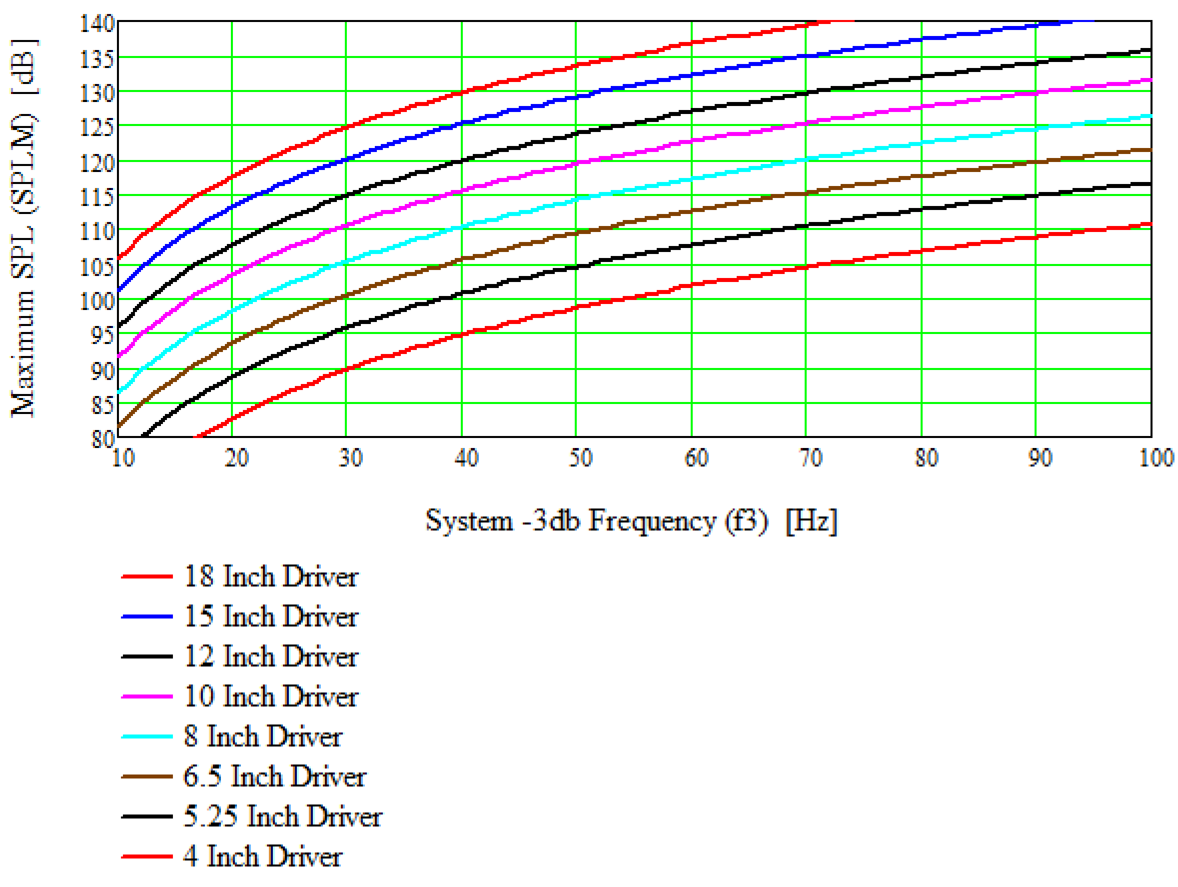

Figure 6: Maximum System SPL vs System f3 for various Driver Sizes (from Equation 18)

Figure 6 is a plot of Equation 18 with the driver based maximum SPL plotted against the system f3. Once again, this is a lot of information. Figure 6 makes it really clear just how much driver size matters; if you want to play louder at a given frequency, you need to go bigger. Figure 6 gives the designer an idea of what to expect from different driver sizes, and how output level will vary with f3 for a given driver size.

Per the logic of Figure 4 you can add about 6 dB to figure 6 for a subwoofer driver, and take off about 6 dB for a pro audio driver. The combination of Figures 5 and 6 give the designer a very quick idea of the port and driver sizes needed to meet a given performance target.

Equations 14 and 18 and their plots in Figures 5 and 6 tell us a lot about a speaker system. If you know the driver and port size you have a good idea of how loud/low it will play. What is a little unexpected is that Equations 14 and 18 don't say anything about the size of the box or the required input power. They just tell you what to expect from given port and driver sizes, but the box size and input power don't show-up. They clearly matter, we just have to go elsewhere to find them.

The Iron Law

Sorry, but I just love that term! It's so in your face. It leaves no doubt that, whatever it says, you are not going to get around it! The term "The Iron Law" dates to work in the 1950's by Josef Hofmann, the "H" in KLH. He observed that with the new "Acoustic Suspension" (ie: Sealed) designs they were developing there was a relationship between the size of the box, the lowest frequencies it could play, and the efficiency. If you want a small box to play low frequencies it will be inefficient: It's "The Iron Law"!

Small quantified the Iron Law into a much more universal and useful form when he developed the equations for both sealed and ported systems. His equation for the acoustic efficiency of a ported system is:

$$\eta(fs, Vas, Qes) = \frac{4*\pi^2}{c^3} * \frac{fs^3 * Vas}{Qes} \qquad \text{EQ(19)}$$

Where: η is the system efficiency
fs is the driver resonant frequency
Vas is the "equivalent volume" of the driver
Qes is the driver electrical Q
c is the speed of sound (343.2 m/s)

Equation 19 is a quantifiable version of The Iron Law applied to a ported speaker. But it is not very useful for our purposes without a little modification. Here's another place where our assumption of a B4 system starts to pay off. If we assume we have a modern low-loss system and choose a box loss of Qbl=7 and a woofer mechanical damping of Qms=3.0 we will end-up with the B4 response requiring:

• Qbl = 7.0	Where: Qbl is the Q of the box due to losses
• Qms = 3.0	Qms is the mechanical Q of the driver
• fb = f3	fb = box tuning frequency [Hz]
• fs = f3	fs = driver resonant frequency [Hz]
• Qts = 0.4048	Qts = Driver total Q
• Qes = 0.4679	Qes = Driver electrical Q
• Vb = 0.942*Vas	Vb = Box volume [m^3]
• Le = 0	Le = Voice coil inductance [H]

If we sub all of the above into Equation 19 we end up with the efficiency of our typical B4 system as:

$$\eta(f3, Vb) = 2.2 * 10^{-6} * f3^3 * Vb \qquad \text{EQ(20)}$$

Equation 20 is the fraction of acoustical power put into the air by the system for one watt of electrical power input to the system. Put in 1 watt and you get the energy shown by Equation 20. Put in 100 watts and you get 100 times the acoustic energy shown by Equation 20. It scales linearly. Adding Pe as the input electrical power we therefore arrive at:

$$Pa(f3, Vb, Pe) = 2.2 * 10^{-6} * f3^3 * Vb * Pe \quad \text{Where: Pa is the acoustic power [watts]} \qquad \text{EQ(21)}$$

We now know the acoustic power generated, but are really interested in SPL. The relationship between acoustic power and SPL at one meter from a piston on an infinite plane is:

$$SPL(Pa) = 20 * \log\left(\frac{\sqrt{\frac{Pa*\rho*c}{2*\pi}}}{2*10^{-5}}\right) \qquad \text{EQ(22)}$$

Combining Equations 21 and 22 we get the SPL produced by our B4 system as:

$$SPL(f3, Vb, Pe) = 55.62 + 20 * log\sqrt{f3^3 * Vb * Pe}) \qquad \text{EQ(23)}$$

We now have our sought after relationship between SPL, box size, and power. But it is still is not in its most useful form. We want to set Equation 23 equal to Equation 18 that defined the maximum SPL based on the driver. Doing so and then solving for the power gives us what we are looking for:

$$Pe(f3, d, Vb) = 7.76 * 10^{-7} * \left(\frac{\left(\frac{d^3}{2}+2*d^2\right)^2 * f3}{Vb}\right) \qquad \text{EQ(24)}$$

Equation 24 is what we’ve been working toward. It is what is going to help us constrain our B4 design to a given box size and required input power.

This concludes the main analysis portion of the paper. Equations 14, 18, and 24 tell the story we set out to tell. We'll follow-up with an example to see how this all works together. We'll also address some practical considerations that will make it clear that not all theoretically allowable solutions to our equations are practicably possible. We will determine the cutoff where a desirable design that is not practicable with a port can be easily realized by using a PR. Finally, a set of Excel spreadsheets will be introduced that automates the calculations for the user.

Design Example

The design process is pretty simple. It starts with the designer knowing what they want to accomplish in terms of maximum SPL and system f3. Use Figure 6 (or Equation 18) to determine the driver size needed to meet the design requirements. Then substitute the f3 and driver size into Equation 24 to plot the options for box size versus input power that can realize the design. The designer is theoretically able to choose any box size they want, the only restraint being that smaller boxes require more power.

The design example will be for a system that would be a good solution for a normal domestic living space. An f3 of 30 Hz is low enough to capture the majority of music content, and a max SPL of 105 dB is loud enough to have satisfying peak levels in a midsized room. Looking at Figure 5 we see that the line for an 8 inch driver passes through the intersection of 30 Hz and 105.3 dB, so it is the smallest driver size likely to meet our needs. Based on this we have:

f3 = 30 Hz
SPLM = 105.3 dB
d = 8 inches

We can now substitute f3 and d into Equation 24 to see our options for box size and input power. Doing so results in the following plot:

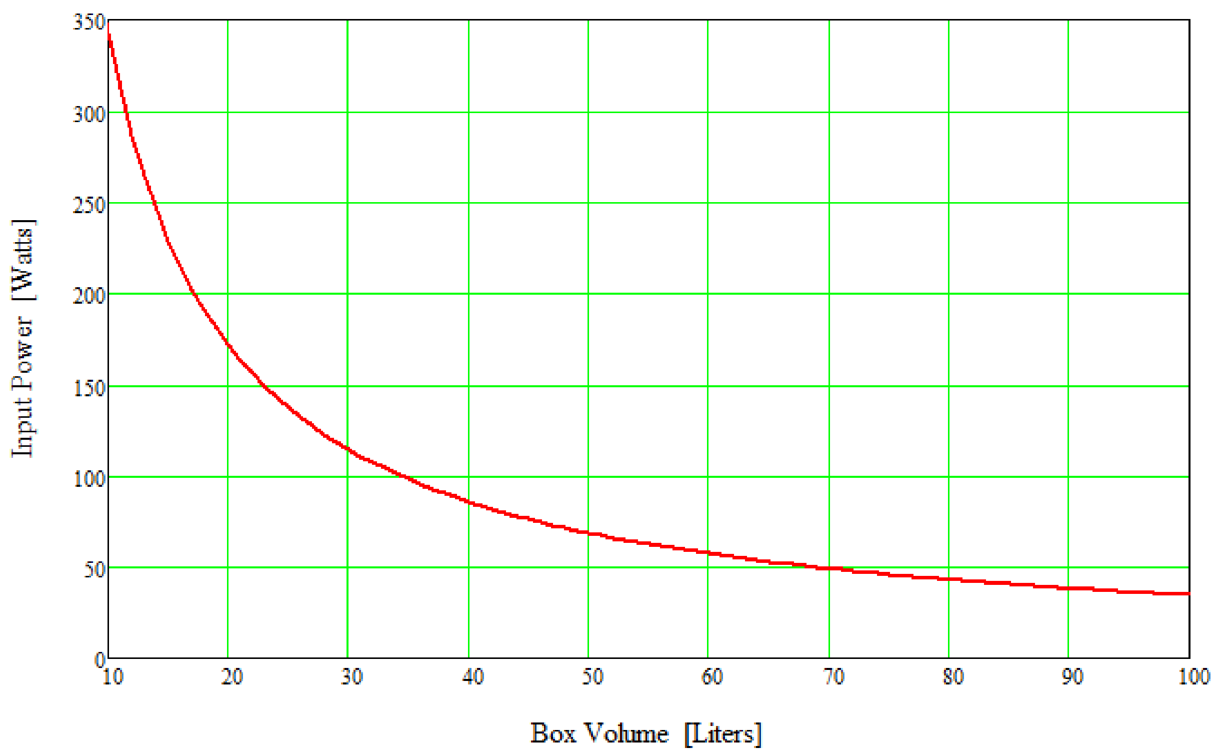

Figure 7: Input power versus box size for f3 = 30Hz and d = 8 inches. System max SPL is 105.3 dB

Figure 7 shows that we can choose any box volume we want, but that as the box gets smaller, the input power goes up dramatically. This is the real-world manifestation of The Iron Law. Our design is nearly complete. If we say we have 175 watts available (171.6 to be precise), we can choose a 20 liter box and the system is pretty well quantified. We just have to figure out the port. We can pick the port diameter visually from Figure 4, or we can rearrange Equation 14 to solve explicitly for the port diameter as:

$$Dp(f3, SPLM) = \sqrt{\frac{10^{\frac{SPLM-58.14}{20}}}{f3}} \qquad \text{EQ(25)}$$

Inserting 30 Hz and 105.31 dB (the exact value from Equation 18) into Equation 24 yields a required port diameter of 2.76 inches. This is the minimum port diameter that can support our system requirement with a maximum port air velocity of 20 m/s. A larger diameter port is always possible, but a smaller diameter port will limit the maximum system SPL per our assumptions.

The last thing we don't know for our quick design is the port length. We know that a requirement of the B4 alignment is that the box tuning frequency equals the f3 equals the driver resonant frequency (fb = f3 = fs). The equation for calculating the length of a circular port is:

$$Lp(f3, Vb, Dp) = 59.5 * \frac{Dp^2}{Vb*f3^2} - 0.73 * Dp \qquad \text{EQ(26)}$$

Where: LP is port length in inches
Dp is port dimeter in inches
Vb is box volume in m^3

Applying equation 26 to our system we arrive at a port length of 23.2 inches. The overall proposed system is then:

- 8 inch driver
- 2.76 inch dimeter x 23.2 inch long port
- 20 liter internal volume box
- f3 of 30Hz
- Max SPL of 105.3 dB
- 172 watts of input power required

That would be the basis for a pretty good system!

There are still some things we don't know. They fall under the practical considerations. Can we fit the port in the box? We know the driver size and f3, but we don't know the driver's required T/S parameters. Note that every combination of f3, SPL, and Vb for a given driver size requires different T/S parameters. Our design was based on a target performance, not a given driver. The driver required to match that performance has yet to be defined.

Practical Considerations

The first practical consideration is if the port will fit the box. Equation 26 tells us that the port must get longer as the box gets smaller in order to maintain the tuning. It makes sense that there must be some box volume below which the port will be too long to fit inside the box.

There is no universally accepted relationship between box volume and how long of port will fit, so we will create our own. Conventional designs usually have the port opening on either the front or rear panel. If the box were a cube (not generally a good idea), each dimension would be the cube root of the box volume. So a good starting point would be to say the maximum port length is the cube root of the box volume. But the inside of the port must clear the back wall of the box, and one port diameter of clearance is usually recommended to avoid air flow problems. So this says that a good estimate for the maximum port length is the cube root of the box volume minus one port diameter.

$$Lpmax(Vb, Dp) = \frac{Vb^{\frac{1}{3}}}{0.0254} - Dp \qquad \text{EQ(27)}$$

Where: Lpmax is the maximum port length in inches
Vb is box volume in liters
Dp is port diameter in inches

If we set Equations 26 and 27 equal to each other and rearranging we get:

$$39.37 * Vb^{\frac{4}{3}} - 0.27 * Dp * Vb = 59.53 * \left(\frac{Dp^2}{f3^2}\right) \qquad \text{EQ(28)}$$

Solving Equation 28 for Vb will tell us the smallest box volume that will fit our required port length. In our design example we had an f3 of 30 Hz and a port diameter of 2.76 inches. Subbing these into Equation 28 and solving for the box volume gives Vb as 0.0397 m^3 (39.7 Liters). (Note that Equation 28 has no simple algebraic solution, so we must use an iterative solver. Fortunately we will have this at our disposal when we get to the spreadsheet!)

In our design example we chose a 20 liter box, and it required a 23 inch long port. Equation 28 tells us that this combination is not going be to very realizable if we want to use a conventional design. The port will be too long to fit in the box. We can therefore use Equation 28 to inform the decision we make when choosing the box size and power from the equivalent of Figure 7 (or Equation 24) for any given design.

We are always free to choose a box volume larger than the limit imposed by Equation 28, and we can meet our f3 and SPLM goals. Choosing a box size smaller than Equation 28 says we will either have to:

A) Use a smaller diameter port to maintain the tuning and preserve f3, sacrificing SPLM.

or

B) Use a shorter port to maintain the SPLM, forcing f3 higher.

Neither of these options lets us meet our performance target if we choose a box smaller than Equation 28 recommends.

If you compare the driver size, box volume, and port diameter this paper recommends to commercial systems, you'll see they typically use a smaller port than recommended. This will limit the maximum system SPL to less than it might otherwise attain. Given the choices, it probably makes sense to go with a smaller diameter port in order to reduce the box size and accept that at loud volumes the port will limit the system. Ports don't instantly stop working at our assumed maximum of 20 m/s air velocity, and the onset of compression and distortion is usually gradual. A smaller than recommended port is probably the best option for shrinking the box size if a port is used, but it does have a performance cost!

Going back to our design example, we'll replot Figure 7 with the box volume we found by solving Equation 28.

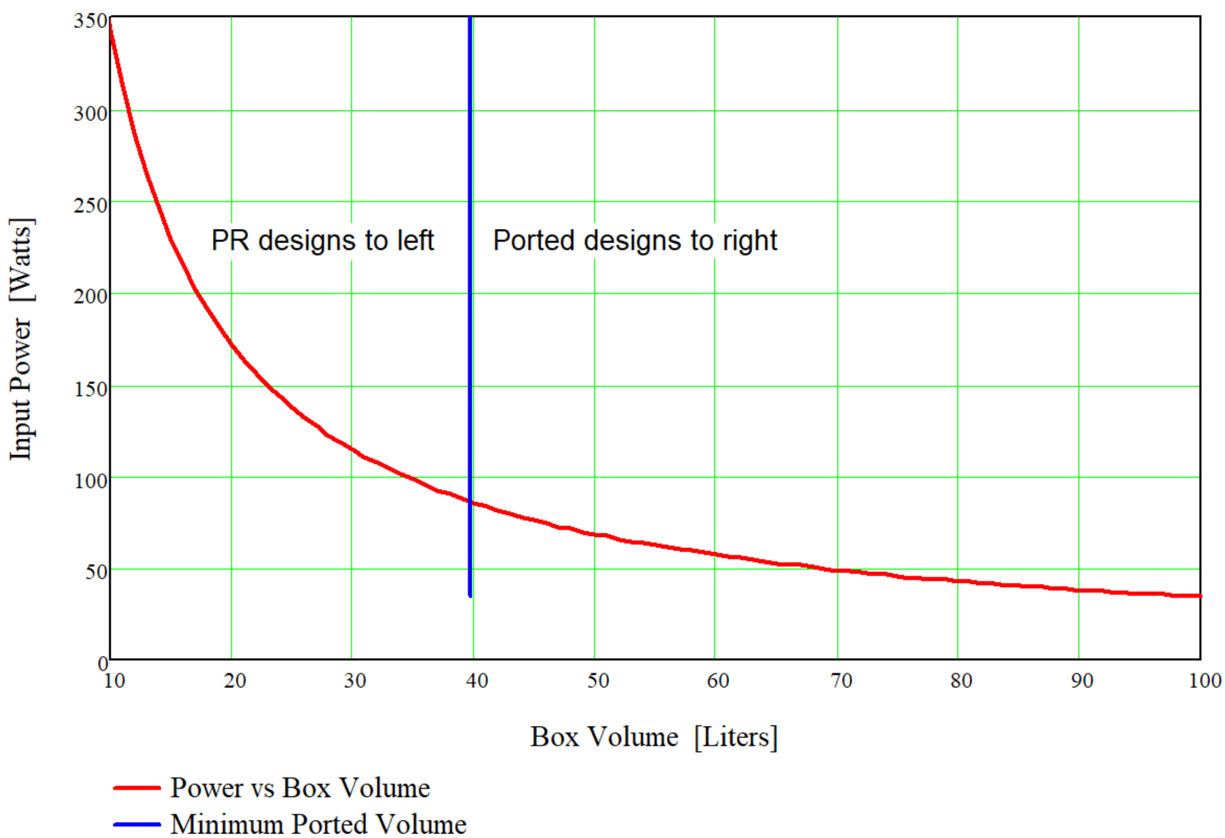

Figure 8: Input power versus box size for f3 = 30Hz and d = 8 inches. Minimum volume to fit port is 40L.

The vertical line in Figure 8 represents the 39.7 liter box volume that we determined to be the minimum box size that would fit the required 2.76 inch diameter port and tune the box to the required 30 Hz. Choosing a box smaller than this will impact the performance if we use a port. However, if we use a PR we can overcome the port limitations and open-up the design space to the left of the vertical line.

Figure 8 shows us in very clear terms when we are OK using a port and when a PR is necessary to achieve our target performance. Our design example has pretty good performance with an f3 of 30Hz and a maximum SPL of 105dB, and it only takes about 85 watts. But the box volume of 40 liters is pretty large. This would be a very large "bookshelf" or a modestly large tower speaker. Power is cheap in the modern world, and drivers capable of handling peaks of a few hundred watts are not unusual or exotic. Our original goal of a 20 liter box volume would make for a reasonable large bookshelf design with really good performance for the category. Figure 8 says it's very doable with a PR, but not with a port.

What price do we pay if we decide to use a 20 liter ported box in our example and: A) Use the biggest port that will fit in the box while tuning it to 30Hz? B) Maintain the maximum SPL but change the port size such that we achieve the lowest f3 while maintaining the SPL?

> A) To figure this out we have to solve Equation 28 with Vb and f3 as the knowns and Dp as the unknown. Doing so gives:
>
> Vb = 0.020 [m^3]   f3 = 30.00 [Hz]   Dp = 1.76 [in]   SPLM = 97.5 [dB]
>
> The port diameter decreases by a full inch! The new box and port size are both reasonable. We can assess the impact on the maximum SPL by turning to Equation 14 to calculate the maximum SPL that a 1.76 inch dimeter port can produce with an f3 of 30Hz. Doing the calculation gives 97.5 dB. So our decision to decrease the box from 40 liters to 20 liters has cost us 7.8dB of maximum output (105.3-97.5).
>
> B) In this case we would turn to Equation 25 to find the port diameter as a function of f3 with our value of SPLM at 105.31dB. Substituting Equation 25 for Dp(f3) into Equation 28 and iterate for the box volume as a function of f3 gives an f3 of 40.7Hz for a 20 liter box. Equation 26 gives the port length as 8.3 inches.
>
> Vb = 0.020 [m^3]   f3 = 40.74 [Hz]   Dp = 2.37 [in]   SPLM = 105.3 [dB]
>
> So if we want to keep the SPLM of 105.3dB when we shrink the box to 20 liters we have to raise the f3 from 30Hz to nearly 41Hz.

The above two analyses show how to determine the impact of shrinking the box size while still using a port. Again, assuming access to a "good" PR that won’t limit the performance, we could shrink our example box to 20 liter with a PR while maintaining the f3 and SPLM.

Driver Requirements

We already made a few basic assumptions about the driver. In the following it is assumed that the user has specified a driver diameter (d), a voice coil resistance (Re), and a box size (Vb). The development of the equations in the paper assumed no voice coil inductance (Le), a box loss of Qbl=7.0, and a driver mechanical Q of Qms=3.0.

The B4 alignment requires enough other things to uniquely define the driver required to meet any combination of f3, SPLM, and box size. The relationships that define the driver are as follows:

$Sd = 0.00035 * d^2$	Assumed driver area	EQ(29)
$Vas = \frac{Vb}{0.942}$	Required B4 criteria	EQ(30)
$Qts = 0.4048$	Required B4 total Q	EQ(31)
$Qes = 0.468$	Required B4 electrical Q	EQ(32)
$Kd(d, Vb) = 0.0164 * \frac{d^4}{Vb}$	Driver stiffness [N/m]	EQ(33)
$Md(d, f3, Vb) = 4.15 * 10^{-4} * \frac{d^4}{Vb * f3^2}$	Driver moving mass [kg]	EQ(34)
$Dd(d, f3, Vb, Qms) = 0.00261 * \frac{d^4}{Vb * f3 * Qms} = \frac{\sqrt{Kd * Md}}{Qms}$	Driver damping [kg/s]	EQ(35)
$Bl(Re) = \sqrt{\frac{Re * \sqrt{Kd * Md}}{Qes}}$	Driver Bl product	EQ(36)

Substituting the above values into the ported modeling program will result in a system with a B4 frequency response.

Spreadsheets

A series of spreadsheets built with Excel 2010 accompany this white paper. They are described as follows.

100Hz B4 Model.xlsm: This is a version of the Vanatoo Thiele/Small ported loudspeaker model that uses a 4th Order Butterworth alignment (B4). It is intended to let the user see the basic response shape of a B4 system. It is password protected by default.

General Purpose Ported T-S Loudspeaker Model.xlsx: This is basically the unlocked version of the above spreadsheet that lets the user modify inputs to model any ported system. Only certain cells are unlocked by default to keep the user from inadvertently changing parameters that should be fixed. But the password is included for those willing to take the risk.

General Purpose PR T-S Loudspeaker Model.xlsm: This is the Vanatoo Thiele/Small passive radiator model. Only certain cells are unlocked by default to keep the user from inadvertently changing parameters that should be fixed. But the password is included for those willing to take the risk.

SPLM vs f3 and Power vs Vb for Ported B4 System.xlsm: This is from the heart of the White Paper. Macros must be enabled in order to run this model. The first tab of this model lets the user pick a driver size and f3 from a duplicate of Figure 6 and Equation 18 from the paper. The minimum box size to fit the recommended port diameter is solved iteratively using Excel's "Solver" routine (that's the required macro). The driver requirements are calculated and the results are then automatically loaded in the second tab of the worksheet so the user can see the modeled results.